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Glauber theory of final-state interactions in $(e, e'p)$ scattering

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A b s t r a c t

We develop the Glauber theory description of final-state interaction (FSI) in quasielastic $A(e, e'p)$ scattering. The important new effect is an interaction between the two trajectories which enter the calculation of FSI-distorted one-body density matrix and are connected with incoherent elastic rescatterings of the struck proton on spectator nucleons. We demonstrate that FSI distortions of the missing momentum distribution are large over the whole range of missing momenta. Important finding is that incoherent elastic rescatterings of the ejected proton lead to a strong quantum mechanical distortions of both the longitudinal and transverse missing momentum distributions. It is shown that allowance for finite longitudinal size of the interaction region for proton-nucleon collision neglected in the standard Glauber model drastically affects the theoretical predictions at high longitudinal missing momentum. We also find very large corrections to the missing momentum distribution calculated within the local-density approximation.

1 Introduction

In recent years much experimental and theoretical efforts have been directed towards the investigation of the final-state interaction (FSI) effects in quasielastic $A(e,e'p)$ scattering at high Q^2. The quantity which is usually used to characterize the strength of the FSI in a certain kinematical region, D, of the missing energy , E_m, and the missing momentum, $\vec{p}_m$, is the nuclear transparency, $T_A(D)$, defined as the ratio

$$T_A(D) = \frac{\int_D dE_m d^3\vec{p}_m d\sigma(E_m,\vec{p}_m)}{\int_D dE_m d^3\vec{p}_m d\sigma_{PWIA}(E_m,\vec{p}_m)}\,. \tag{1}$$

Here $d\sigma(E_m,\vec{p}_m)$ is the experimentally measured cross section, $d\sigma_{PWIA}(E_m,\vec{p}_m)$ is the theoretical cross section calculated in the plane wave impulse approximation (PWIA) when FSI is not taken into account. The missing energy is defined as $E_m = m_A - m_p - E_{A-1}$, where m_A and m_p are the target nucleus and proton mass, respectively, E_{A-1} is the energy of the residual nucleus. The missing momentum is connected with the virtual photon three-momentum, $\vec{q}$, and the momentum of the ejected proton, $\vec{p}$, $\vec{p}_m = \vec{q} - \vec{p}$.

The strong interactions that the stuck proton undergoes during its propagation through the nuclear medium lead to the deviation of T_A from unity. It is expected [1, 2] however, that, at asymptotically high Q^2, T_A must tend to unity due to the color transparency (CT) phenomenon in QCD [3–5] (for the recent review on CT see ref. [6]). From the point of view of the multiple scattering theory this effect corresponds to a cancellation between the rescattering amplitudes with elastic (diagonal) and inelastic (off-diagonal) intermediate states. These coupled-channel effects only become important at sufficiently high Q^2 and the recent quantum mechanical analysis [7] of $A(e,e'p)$ scattering has shown that CT effect from the off-diagonal contribution to FSI is still very small in the region $Q^2 \lesssim 10$ GeV^2. The energy (Q^2) dependence of the nuclear transparency observed in the NE18 experiment [8] is in agreement with the one predicted in [7]. Thus, there are reasons to expect that in the region $Q^2 \sim 2-10$ GeV^2, which is particularly interesting from the point of view of future high-statistics experiments at CEBAF, FSI in $A(e,e'p)$ scattering will be dominated by elastic rescatterings of the struck proton on the spectator nucleons. In this

region of Q^2 the typical kinetic energy of the struck proton $T_{kin} \approx Q^2/2m_p \gtrsim 1$ GeV is sufficiently large and FSI can be treated in the framework of the standard Glauber model [9]. The purpose of the present paper is the Glauber theory description of the missing momentum distribution in inclusive $(e, e'p)$. We focus on the region of missing momenta $p_m \lesssim k_F$ ($k_F \sim 250$ MeV/c is the Fermi momentum). Such an analysis is interesting for several reasons:

First, understanding the $\vec{p}_m$-dependence of FSI effects is necessary for accurate interpretation of the data from NE18 experiment [8] and from future experiments at CEBAF. The point is that experimentally one measures the $A(e, e'p)$ cross section only in a certain restricted window D in the missing momentum. Because FSI affects the missing momentum distribution as compared to the PWIA case, the absolute value and the energy dependence of the experimentally measured nuclear transparency will be different for different kinematical domains D. Consequently, understanding the p_m and Q^2 dependence of the conventional FSI effects is imperative for disentangling the small CT effects at CEBAF and beyond.

Secondly, the still another CT effect which can be obscured by FSI is an asymmetry of nuclear transparency about $p_{m,z} = 0$ [10, 11, 12] (as usual, we choose the z axis along the virtual photon's three-momentum). The CT induced forward-backward asymmetry increases with Q^2. However, similar forward-backward asymmetry is generated by FSI already at the level of elastic rescatterings of the struck proton from the spectator nucleons. It is a consequence of the nonzero real part of the elastic pN amplitude. The qualitative estimates [6] show that at $Q^2 \lesssim 10$ GeV^2, the FSI-induced asymmetry can overcome the CT-induced effect. For this reason, the interpretation of results from the future CEBAF experiments on the forward-backward asymmetry as a signal for the onset of CT requires the accurate calculation of the missing momentum distribution in the Glauber model.

At last but not the least, the quantitative theory of FSI in quasielastic $(e, e'p)$ scattering is interesting from the point of view of the nuclear physics as well. At high momenta, the single-particle momentum distribution (SPMD) is sensitive to short range NN corre-

lations in nuclei. Because of FSI effects, the experimentally measured missing momentum distribution in inclusive $A(e,e'p)$ scattering may considerably differ from the real SPMD. The recent study [13, 14] has shown that even in light nuclei $(D,^4He)$, in which the probability of FSI is still small, at high missing momenta a nontrivial "interference" of the FSI effects and short range NN correlation takes place. In heavier nuclei, which we study in the present paper, FSI effects turn out to be strong even in the region $p_m \lesssim 300$ MeV/c, where the role of the short range NN correlation is still marginal. Nevertheless our results help to get insight into the role of the FSI at the missing momenta $p_m \sim 300$ MeV/c which are close to the kinematical region where the short range NN correlations become important. Our particularly important finding is that the incoherent rescatterings of the struck proton from the spectator nucleons considerably affect the longitudinal missing momentum distribution as compared to SPMD. They lead to large tails in the missing momentum distribution at high $|p_{m,z}|$. The observed effect is of purely quantum-mechanical origin and defies the classical treatment.

One important finding from our study of FSI a natural applicability limit for the Glauber formalism in the case of $A(e,e'p)$ reaction. It is connected with the finite longitudinal size of the interaction region for the proton-nucleon collisions, which is about the proton radius and is neglected in standard applications of the multiple scattering theory. We show that for this reason the standard Glauber model predicts an anomalously slow decrease ($\propto |p_{m,z}|^{-2}$) of the missing momentum distribution at high longitudinal missing momenta. The physical origin of this anomalous behavior is an incorrect treatment in the Glauber model of the incoherent rescatterings of the struck proton on the adjacent spectator nucleons, when the separation between the struck proton and spectator nucleons is comparable with the proton size. Our estimates show that there can be large uncertainties due the finite-proton size effects at $|p_{m,z}| \gtrsim 500$ MeV/c. In this region of $|p_{m,z}|$, besides the short range NN correlations, the experimentally measured missing momentum distribution becomes sensitive to the finite-proton size effects in FSI, which makes the experimental study of NN correlations much more difficult. It is important

that this novel sensitivity of the missing momentum distribution at high $|p_{m,z}|$ to the proton size does not disappear at high Q^2, and that the same situation takes place for the multiple scattering theory as a whole when inelastic (off-diagonal) rescatterings of the struck proton are included. For this reason the effect must be taken into account in the interpretation of the experimental data in high missing momentum region from future experiments at large Q^2.

Our paper is organized as follows. In section 2 we derive the formulas for calculation of the missing momentum distribution in quasielastic $(e,e'p)$ scattering within the Glauber model. We discuss briefly the generalization of our formalism to the case with inclusion of the CT effects as well. We conclude section 2 with comments on other works on the application of the Glauber model to $(e,e'p)$ reaction. Section 3 is devoted to the detailed comparison of the Glauber model with the optical potential approach. In particular, we discuss the difference between the treatment in these two models of the contribution to the FSI from the incoherent rescatterings of the struck proton in the nuclear medium. In conclusion of section 3 we discuss the formal analogy between the treatment of the FSI effects in the optical potential approach and the Glauber formalism in the case of exclusive $(e,e'p)$ scattering. In section 4 we derive the multiple-scattering series for the transverse missing momentum distribution. We show that $p_\perp$ distribution can be formally represented in a form when all the quantum mechanical distortion effects are contained in the local missing momentum distribution calculated without inclusion of the incoherent rescatterings of the struck proton on the spectator nucleons. In section 5 we discuss in detail the influence of the FSI effects upon the longitudinal missing momentum distribution. We show that the incoherent rescatterings of the struck proton on the spectator nucleons considerably affect the measured in quasielastic $(e,e'p)$ scattering missing momentum distribution at high $|p_{m,z}|$. The qualitative quantum mechanical analysis of this phenomenon is presented. We conclude section 5 by discussion of the applicability limits of the Glauber model. In section 6 we present our numerical results. The summary and conclusions are presented in section 7.

One remark on our terminology is in order: In the present paper we consider $(e, e'p)$ reaction without production of new hadrons. We will use the term "inclusive $(e, e'p)$ scattering" for the processes in which the final state of the residual nucleus is not observed. The term "exclusive $(e, e'p)$ scattering" will be used for the processes in which there is only one knocked out nucleon (the struck proton).

2 FSI and the missing-momentum distribution in Glauber formalism

In the present paper we will restrict ourselves to the case of the mass number of the target nucleus $A \gg 1$. In this case, neglecting the center of mass correlations, we can write the reduced nuclear amplitude of the exclusive process $e + A_i \rightarrow e' + (A-1)_f + p$ in the form

$$M_f = \int d^3\vec{r}_1 ... d^3\vec{r}_A \Psi_f^*(\vec{r}_2, ..., \vec{r}_A)\Psi_i(\vec{r}_1, ..., \vec{r}_A) S(\vec{r}_1, ...\vec{r}_A) \exp(i\vec{p}_m\vec{r}_1)\,. \tag{2}$$

Here Ψ_i and Ψ_f are wave functions of the target and residual nucleus, respectively. The nucleon "1" is chosen to be the struck proton. For the sake of brevity in Eq (2) and hereafter the spin and isospin variables are suppressed. The function $S(\vec{r}_1, ..., \vec{r}_A)$ describes the FSI of the struck proton in the nuclear medium. In the Glauber approximation it is given by

$$S(\vec{r}_1, ..., \vec{r}_A) = \prod_{j=2}^{A} \left[1 - \theta(z_j - z_1)\Gamma(\vec{b}_1 - \vec{b}_j)\right]\,, \tag{3}$$

where $\vec{b}_j$ and z_j are the transverse and longitudinal coordinates of the nucleons and $\Gamma(\vec{b})$ is the familiar profile function of the elastic proton-nucleon scattering. We use for $\Gamma(\vec{b})$ the standard high-energy parameterization

$$\Gamma(b) = \frac{\sigma_{tot}(pN)(1 - i\alpha_{pN})}{4\pi B_{pN}} \exp\left[-\frac{b^2}{2B_{pN}}\right]\,. \tag{4}$$

Here α_{pN} is the ratio of the real to imaginary part of the forward elastic pN amplitude, B_{pN} is the diffractive slope describing the t dependence of the elastic proton-nucleon cross

section

$$\frac{d\sigma_{el}(pN)}{dt} = \left.\frac{d\sigma_{el}(pN)}{dt}\right|_{t=0} \exp(-B_{pN}|t|)\,. \qquad (5)$$

In the Glauber's high-energy approximation, the struck proton propagates along the straight-path trajectory and can interact with the spectator nucleon "j" only provided that $z_j > z_1$, which is an origin of the step-function $\theta(z_j - z_1)$ in the FSI factor (3). Physically it implies that we neglect the finite longitudinal size of the region where the struck proton interacts with the spectator nucleon. The consequences of this assumption will be discussed in section 5.

Our aim is the calculation of the (inclusive) missing momentum distribution, $w(\vec{p}_m)$, or nuclear transparency $T_A(\vec{p}_m)$, in the kinematical conditions when the events of the whole range of E_m are included (in a sense this case corresponds to the experimental situation of "poor-energy resolution"). In this case the missing momentum distribution may be written as

$$w(\vec{p}_m) = \frac{1}{(2\pi)^3}\int dE_m S(E_m, \vec{p}_m)\,, \qquad (6)$$

where $S(E_m, \vec{p}_m)$ is the spectral function taking into account the FSI of the struck proton

$$S(E_m, \vec{p}_m) = \sum_f |M_f(\vec{p}_m)|^2 \delta(E_m + E_{A-1}(\vec{p}_m) + m_p - m_A)\,. \qquad (7)$$

At high Q^2, the sum over all final states of the residual nucleus required for the calculation of the missing momentum distribution from Eqs. (6), (7) can be performed making use of the closure relation

$$\sum_f \Psi_f(\vec{r}_2^{\,\prime}, ..., \vec{r}_A^{\,\prime})\Psi_f^*(\vec{r}_2, ..., \vec{r}_A) = \prod_{j=2}^{A} \delta(\vec{r}_j - \vec{r}_j^{\,\prime}) \qquad (8)$$

Employing Eq. (8) makes it possible to represent $w(\vec{p}_m)$ in the following form

$$w(\vec{p}_m) = \frac{1}{(2\pi)^3}\int d^3\vec{r}_1 d^3\vec{r}_1^{\,\prime} \prod_{j=2}^{A} d^3\vec{r}_j \exp[i\vec{p}_m(\vec{r}_1 - \vec{r}_1^{\,\prime})]$$
$$\times\Psi_i(\vec{r}_1, \vec{r}_2, ..., \vec{r}_A)\Psi_i^*(\vec{r}_1^{\,\prime}, \vec{r}_2, ..., \vec{r}_A)S(\vec{r}_1, \vec{r}_2, ..., \vec{r}_A)S^*(\vec{r}_1^{\,\prime}, \vec{r}_2, ..., \vec{r}_A)\,. \qquad (9)$$

Thus, from the point of view of the nuclear physics the calculation of the missing momentum distribution in quasielastic $(e, e'p)$ scattering is reduced to the evaluation of the

ground state expectation value for a special many-body operator

$$w(\vec{p}_m) = \langle \Psi_i | U(\vec{p}_m) | \Psi_i \rangle \,, \tag{10}$$

where

$$\langle \vec{r}_1^{\,'}, ..., \vec{r}_A^{\,'} | U(\vec{p}_m) | \vec{r}_1, ..., \vec{r}_A \rangle = \frac{1}{(2\pi)^3} \exp[i\vec{p}_m(\vec{r}_1 - \vec{r}_1^{\,'})] \prod_{j=2}^{A} \delta(\vec{r}_j^{\,'} - \vec{r}_j) S^*(\vec{r}_1^{\,'}, ..., \vec{r}_A^{\,'}) S(\vec{r}_1, ..., \vec{r}_A) \,. \tag{11}$$

The peculiarity of the operator U (11) is that it distorts the target nucleus wave function in the variables $\vec{r}_2, ..., \vec{r}_A$ only when some of $\vec{r}_i$ are close to at least one of the two straight-path trajectories beginning from the points $\vec{r}_1$ and $\vec{r}_1^{\,'}$, which arise after taking the square of the reduced nuclear matrix element (2). The crucial point of the further analysis is that FSI generates short range interaction between these two trajectories, which will be one of the main factors on the distortion of missing momentum distribution as compared to SPMD.

It is worth noting at this point that up to now we did not use the concrete form of the Glauber model attenuation factor (3). It is clear that generalization of Eq. (9) to the case when the CT effects are included is reduced to the following replacement

$$S(\vec{r}_1, \vec{r}_2, ..., \vec{r}_A) \Rightarrow \frac{\langle p | \hat{S}_{3q}(\vec{r}_1, \vec{r}_2, ..., \vec{r}_A) | E \rangle}{\langle p | E \rangle} \,. \tag{12}$$

Here $\hat{S}_{3q}(\vec{r}_1, \vec{r}_2, ..., \vec{r}_A)$ is an operator describing the evolution of the three-quark wave function of the struck proton during its propagation in the nuclear medium, $|E\rangle$ is an three-quark wave function which describes the state of the proton after absorption of the virtual photon at point $\vec{r}_1$. In terms of the electromagnetic current operator $\hat{J}_{em}$, the ejectile wave function is expressed as [11]

$$|E\rangle = \hat{J}_{em}(Q)|p\rangle = \sum_i |i\rangle\langle i|J_{em}(Q)|p\rangle = \sum_i G_{ip}(Q)|i\rangle \,, \tag{13}$$

where $G_{ip}(Q) = \langle i|J_{em}(Q)|p\rangle$ includes the electromagnetic form factor of the proton as well as all transition form factors for the electroexcitation of the proton $e + p \to e' + i$. In the case of the nonrelativistic oscillator quark model the evolution operator S_{3q} can

be computed using the path integral technique [15, 16, 17]. It is possible to evaluate this operator in the multiple scattering approach as well [7]. In the present paper we restrict ourselves to the calculation of the missing momentum distribution in the Glauber approximation. The analysis taking into account the CT effects will be presented elsewhere.

In the above analysis we factored out the electromagnetic current matrix elements of the struck proton. In doing so, we followed the usual tradition [18] of neglecting possible departures of these matrix elements from the PWIA matrix elements, which may emerge because of the off-mass shell effects [19] induced by FSI. Under this assumption the (local) nuclear transparency as a function of the missing momentum can be written as

$$T_A(\vec{p}_m) = w(\vec{p}_m)/n_F(\vec{p}_m)\,, \tag{14}$$

where $n_F(\vec{p})$ is the SPMD that can be expressed through the one-body nuclear density matrix, $\rho(\vec{r},\vec{r}^{\,'})$,

$$n_F(\vec{p}) = \frac{1}{(2\pi)^3}\int d^3\vec{r}d^3\vec{r}^{\,'}\rho(\vec{r},\vec{r}^{\,'})\exp[i\vec{p}(\vec{r}-\vec{r}^{\,'})]\,. \tag{15}$$

The normalization of $n_F(\vec{p})$ is as follows

$$\int d^3\vec{p}n_F(\vec{p}) = 1\,.$$

Integration of the missing momentum distribution $w(\vec{p}_m)$ over the whole region of $\vec{p}_m$ gives the integral nuclear transparency

$$\int d^3\vec{p}_m w(\vec{p}_m) = T_A\,. \tag{16}$$

Using Eqs. (9), (16) one can obtain the following expression for T_A

$$T_A = \int\prod_{j=1}^{A} d^3\vec{r}_j|\Psi_i(\vec{r}_1,...,\vec{r}_A)|^2|S(\vec{r}_1,...,\vec{r}_A)|^2\,. \tag{17}$$

Besides the distribution $w(\vec{p}_m)$, which is normalized to T_A, we will use in the present paper the missing momentum distribution $n_{eff}(\vec{p}_m)$, which is normalized to unity

$$n_{eff}(\vec{p}_m) = w(\vec{p}_m)/T_A\,. \tag{18}$$

Equations (3), (9) can be used as a basis for the calculation of the nuclear transparency in reaction $(e, e'p)$ within the Glauber model. Evidently, even without taking into account the CT effects, evaluation of the nuclear transparency (especially if we are interested in the $\vec{p}_m$ dependence of $T_A(\vec{p}_m)$ or $w(\vec{p}_m)$) is quite an involved problem. In this communication we confine ourselves to an evaluation of FSI effects at moderate missing momenta $p_m \lesssim k_F$, where the simple shell model is well known to give good description of SPMD and short-range NN correlations effects are marginal [20]. In the opposite to that, at high missing momenta $p_m \gtrsim k_F$, there emerges a complicated pattern of the interference effects between the short range NN correlations in the nucleus wave function [21, 22, 20] and FSI of the struck proton [13, 14]. Furthermore, even at large p_m, FSI effects turn out to be numerically substantially larger than NN correlation effects [13, 14, 23]. Therefore, as far as the salient features of FSI, in particular the understanding of the rôle of interaction between the two trajectories in FSI factor, are concerned, it is reasonable to use a simple independent particle nuclear shell model for calculation of the missing momentum distribution in the region $p_m \lesssim k_F$.

In this case, making use of the Slater determinant form of the shell model wave function we can write the product of the wave functions appearing in Eq. (9) in the form

$$\Psi_i(\vec{r}_1, \vec{r}_2, ..., \vec{r}_A)\Psi_i^*(\vec{r}_1^{\,\prime}, \vec{r}_2, ..., \vec{r}_A) = \frac{1}{Z}\sum_n \phi_n(\vec{r}_1)\phi_n^*(\vec{r}_1^{\,\prime})\rho_n(\vec{r}_2, ..., \vec{r}_A)$$
$$+\frac{1}{Z}\sum_{n\neq m} \phi_n(\vec{r}_1)\phi_m(\vec{r}_1^{\,\prime})\rho_{nm}(\vec{r}_2, ..., \vec{r}_A)\,. \tag{19}$$

Here Z is the number of protons in the target nucleus, $\phi_n(\vec{r})$ are single-particle shell model proton wave functions. In the right hand side of Eq. (19) we separated the sum over the struck proton states into the diagonal part (the first term) and nondiagonal one (the second term). The diagonal, $\rho_n(\vec{r}_2, ..., \vec{r}_A)$, and nondiagonal, $\rho_{nm}(\vec{r}_2, ..., \vec{r}_A)$, $(A-1)$-body distributions introduced in Eq. (19) are given by

$$\rho_n(\vec{r}_2, ..., \vec{r}_A) = \Psi_{i,n}(\vec{r}_2, ..., \vec{r}_A)\Psi_{i,n}^*(\vec{r}_2, ..., \vec{r}_A)\,, \tag{20}$$

$$\rho_{nm}(\vec{r}_2, ..., \vec{r}_A) = \Psi_{i,n}(\vec{r}_2, ..., \vec{r}_A)\Psi_{i,m}^*(\vec{r}_2, ..., \vec{r}_A)\,, \tag{21}$$

where $\Psi_{i,n}$ is the $(A-1)$-body wave function which describes the system of $(A-1)$ nucleons obtained after removing of the proton in the state n from the target nucleus. The $(A-1)$-body nuclear density of this system, given by the function ρ_n, is normalized to unity

$$\int \prod_{j=2}^{A} d^3\vec{r}_j \rho_n(\vec{r}_2, ..., \vec{r}_A) = 1\,. \tag{22}$$

In the case of the nondiagonal distribution ρ_{nm} (21) the corresponding integral over the coordinates $2-A$ is equal zero. However, due to the presence of the Glauber attenuation factors, the contribution to the integral over the coordinates of the spectator nucleons in Eq. (9), related to the last term in the product of the wave function (19), does not vanish. None the less, making use of the random phase approximation one can show that the contribution to the missing momentum distribution related to the nondiagonal part of the product of the target nucleus wave functions is suppressed by the factor $1/A$ in a comparison with the one from the first term in Eq. (19). Thus, to the leading order in $1/A$, the calculation of the missing momentum distribution can be performed keeping in Eq. (19) only the diagonal term.

To proceed with the calculation of the missing momentum distribution we will neglect influence of the Fermi correlations on the $(A-1)$-body nuclear density ρ_n, and approximate it by the factored form (notice that to the leading order in $1/A$ the dependence of ρ_n on index n can be neglected as well)

$$\rho_n(\vec{r}_2, ..., \vec{r}_A) \approx \prod_{j=2}^{A} \rho_A(\vec{r}_j)\,, \tag{23}$$

where $\rho_A(\vec{r})$ is the nucleon nuclear density normalized as

$$\int d^3\vec{r} \rho_A(\vec{r}) = 1\,.$$

The fact that the factored approximation for the many-body nuclear density is a very good one for the purpose of the calculation of the Glauber model attenuation factor for the case of soft hadron nucleus scattering is well known for a long time (for an extensive review on hA scattering see [24], the correlation effect in the integrated nuclear transparency T_A was discussed in [25, 26]).

Making use of Eqs. (19), (23) we can represent the missing momentum distribution (9) in the form

$$w(\vec{p}_m) = \frac{1}{(2\pi)^3}\int d^3\vec{r}_1 d^3\vec{r}_1^{\,'} \rho(\vec{r}_1, \vec{r}_1^{\,'})\Phi(\vec{r}_1, \vec{r}_1^{\,'}) \exp[i\vec{p}_m(\vec{r}_1 - \vec{r}_1^{\,'})]\,, \tag{24}$$

which only differs from the SPMD for the presence of the FSI factor

$$\Phi(\vec{r}_1, \vec{r}_1^{\,'}) = \int \prod_{j=2}^{A} \rho_A(\vec{r}_j) d^3\vec{r}_j S(\vec{r}_1, \vec{r}_2, ..., \vec{r}_A) S^*(\vec{r}_1^{\,'}, \vec{r}_2, ..., \vec{r}_A)\,, \tag{25}$$

which describes the FSI-distortion of the one-body shell model proton density matrix

$$\rho(\vec{r}, \vec{r}^{\,'}) = \frac{1}{Z}\sum_n \phi_n(\vec{r})\phi_n^*(\vec{r}^{\,'})\,. \tag{26}$$

Eq. (24) is a counterpart of the conventional formula (15) for SPMD. Due to the dependence of FSI factor $\Phi(\vec{r}_1, \vec{r}_1^{\,'})$ on $\vec{r}_1$ and $\vec{r}_1^{\,'}$, the normalized missing momentum distribution (18) does not coincide with the Fermi distribution. The FSI factor (25) can not be represented in a factored form in the variables $\vec{r}_1$ and $\vec{r}_1^{\,'}$. In particular, because of FSI the function $w(\vec{p}_m)$ is not isotropic one. We remind that our approach only is applicable in the region of relatively small missing momenta $p_m \lesssim k_F$. We postpone the analysis of the high missing momentum region for further publications.

By using formula (3) we obtain the closed analytical expression for FSI factor (25)

$$\begin{aligned}\Phi(\vec{r}_1, \vec{r}_1^{\,'}) = \Big[1 &- \frac{1}{A}\int d^2\vec{b}\,\Gamma(\vec{b}_1 - \vec{b}) t(\vec{b}, z_1) - \frac{1}{A}\int d^2\vec{b}\,\Gamma^*(\vec{b}_1^{\,'} - \vec{b}) t(\vec{b}, z_1^{'}) \\ &+ \frac{1}{A}\int d^2\vec{b}\,\Gamma^*(\vec{b}_1^{\,'} - \vec{b})\Gamma(\vec{b}_1 - \vec{b}) t(\vec{b}, max(z_1, z_1^{'})\Big]^{A-1}\,,\end{aligned} \tag{27}$$

here we introduced the partial optical thickness function

$$t(\vec{b}, z) = A\int_z^\infty d\xi \rho_A(\vec{b}, \xi)\,. \tag{28}$$

The FSI factor can further be simplified exploiting the fact that the partial optical thickness $t(\vec{b}, z)$ is a smooth function of the impact parameter $\vec{b}$ as compared to the nuclear profile function $\Gamma(\vec{b})$. Then, to the zeroth order in the small parameter B_{pN}/R_A^2 (R_A

is the nucleus radius) the result of integration over the impact parameter $\vec{b}$ in the terms $\propto \Gamma(\Gamma^*)$ in Eq. (27) is given by

$$\int d^2\vec{b}\Gamma(\vec{b}_1-\vec{b})t(\vec{b},z_1) \approx \frac{\sigma_{tot}(pN)(1-i\alpha_{pN})}{2}t(\vec{b},z)\,. \qquad (29)$$

In the same approximation for the term $\propto \Gamma^*\Gamma$ in the brackets in Eq. (27) we have

$$\int d^2\vec{b}\Gamma^*(\vec{b}_1'-\vec{b})\Gamma(\vec{b}_1-\vec{b})t(\vec{b},z) \approx \eta(\vec{b}_1-\vec{b}_1')\sigma_{el}(pN)t(\frac{1}{2}(\vec{b}_1+\vec{b}_1'),z)\,, \qquad (30)$$

where the function $\eta(\vec{b})$ is given by

$$\eta(\vec{b}) = \frac{\int d^2\vec{\Delta}\Gamma^*(\vec{b}-\vec{\Delta})\Gamma(\vec{\Delta})}{\int d^2\vec{\Delta}|\Gamma(\vec{\Delta})|^2}$$
$$= \frac{1}{\pi\sigma_{el}(pN)}\int d^2\vec{q}\frac{d\sigma_{el}(pN)}{dq^2}\exp(i\vec{q}\vec{b}) = \exp\left[-\frac{\vec{b}^2}{4B_{pN}}\right] \qquad (31)$$

(We checked that in the case of the nuclear mass number $A \gtrsim 10$ the corrections to the Eqs. (29), (30) connected with the neglected higher order terms in the ratio B_{pN}/R_A^2 lead to the corrections in the final numerical predictions for the missing momentum distribution and T_A which do not exceed 1-3%.) Finally, making use of Eqs. (27), (29), (30) and exponentiating which is a good approximation at $A \gg 1$, we arrive at the following expression for the FSI factor (25)

$$\Phi(\vec{r}_1,\vec{r}_1') = \exp\left[-\frac{1}{2}\sigma_{tot}(pN)(1-i\alpha_{pN})t(\vec{b}_1,z_1) - \frac{1}{2}\sigma_{tot}(pN)(1+i\alpha_{pN})t(\vec{b}_1',z_1') \right.$$
$$\left. +\eta(\vec{b}_1-\vec{b}_1')\sigma_{el}(pN)t(\frac{1}{2}(\vec{b}_1+\vec{b}_1'),max(z_1,z_1'))\right]\,. \qquad (32)$$

Below we will refer to the first two terms in the exponent in Eq. (32) as $\Gamma(\Gamma^*)$terms , and the last one as $\Gamma^*\Gamma$ term. Notice that, were it not for the $\Gamma^*\Gamma$ terms in the exponent, the FSI factor would have factored into the two independent attenuation factors which only depend on $\vec{r}$ and $\vec{r}\,'$, respectively. The $\Gamma^*\Gamma$ term introduces an interaction between the two trajectories, which is a steep function of $|\vec{b}_1-\vec{b}_1'|$. This interaction substantially affects the observed missing momentum distribution and shall be of major concern in this paper.

Besides the three-dimensional distribution $w(\vec{p}_m)$ we will consider $p_{m,z}$ integrated $\vec{p}_{m\perp}$ distribution, $w_{\perp}(\vec{p}_{m\perp})$, and $\vec{p}_{m\perp}$ integrated $p_{m,z}$ distribution, $w_z(p_{m,z})$. Performing the integration of the distribution $w(\vec{p}_m)$ given by Eq. (24) over transverse and longitudinal component of the missing momentum one can obtain for $p_{m,z}$ and $p_{m\perp}$ distributions

$$w_z(p_{m,z}) = \frac{1}{2\pi}\int d^2\vec{b}dzdz^{'}\rho(\vec{b},z,\vec{b},z^{'})\Phi_z(\vec{b},z,z^{'})\exp[i\vec{p}_{m,z}(z-z^{'})]\,, \tag{33}$$

$$w_{\perp}(\vec{p}_{m\perp}) = \frac{1}{(2\pi)^2}\int d^2\vec{b}d^2\vec{b}^{'}dz\rho(\vec{b},z,\vec{b}^{'},z)\Phi_{\perp}(\vec{b},\vec{b}^{'},z)\exp[i\vec{p}_{m\perp}(\vec{b}-\vec{b}^{'})]\,, \tag{34}$$

where

$$\Phi_z(\vec{b},z,z^{'}) = \exp\Big[-\frac{1}{2}\sigma_{tot}(pN)(1-i\alpha_{pN})t(\vec{b},z) - \frac{1}{2}\sigma_{tot}(pN)(1+i\alpha_{pN})t(\vec{b},z^{'}) + \sigma_{el}(pN)t(\vec{b},max(z,z^{'})\Big]\,, \tag{35}$$

$$\Phi_{\perp}(\vec{b},\vec{b}^{'},z) = \exp\Big[-\frac{1}{2}\sigma_{tot}(pN)(1-i\alpha_{pN})t(\vec{b},z) - \frac{1}{2}\sigma_{tot}(pN)(1+i\alpha_{pN})t(\vec{b}^{'},z) + \eta(\vec{b}-\vec{b}^{'})\sigma_{el}(pN)t(\frac{1}{2}(\vec{b}+\vec{b}^{'}),z)\Big]\,. \tag{36}$$

Eqs. (24), (32), (33)–(36) form a basis for our evaluations of the missing momentum distribution (and $T_A(\vec{p}_m)$) in quasielastic $(e,e'p)$ scattering in the Glauber model. The three-dimensional distribution (24), as it was mentioned in section 1, is particularly interesting from the point of view of using it for an accurate comparison of the theoretical predictions with experimental data on the nuclear transparency obtained for a certain kinematical domain D, when the nuclear transparency $T_A(D)$ is defined according to Eq. (1). In the present paper we will make for the first time such a comparison of the Glauber model predictions with the data from the NE18 experiment [8].

The formalism presented in this section is rather simple and is based upon the same ideas as the original Glauber approach to the hadron-nucleus interactions at small momentum transfer [9]. We gave a very detailed derivation mostly for the reason that in the current literature there exist discussions of the missing momentum distribution in reaction $(e,e'p)$ within the same Glauber model, which incorrectly treat the important

effect of interaction between the two trajectories [27, 28]. For instance, the authors of ref.[27] neglect the dependence on the variable $\Delta\vec{r}_1 = (\vec{r}_1 - \vec{r}_1^{\,\prime})$ in their counterpart of our FSI factor (32) and put in it

$$\vec{r}_1 = \vec{r}_1^{\,\prime} = (\vec{r}_1 + \vec{r}_1^{\,\prime})/2\,. \tag{37}$$

Doing so, they completely missed the rapid dependence on $\Delta\vec{r}_1$ of the $\Gamma^*\Gamma$ term in the exponent of FSI factor. Further, using the Negele-Vautherin local density approximation (LDA) [29] for one-body density matrix (we comment more on this approximation below)

$$\rho(\vec{r},\vec{r}^{\,\prime}) = \rho_A(\frac{1}{2}(\vec{r}+\vec{r}^{\,\prime}))W(\vec{r}-\vec{r}^{\,\prime}) \tag{38}$$

(here $W(\vec{r}-\vec{r}^{\,\prime})$ is the Fourier transform of the Fermi distribution), Kohama et al. find the missing momentum distribution which is proportional to SPMD Fermi momentum distribution. It is clear that in doing so they missed all the distortion effects which, as we shall demonstrate below, are quite strong. The same criticism is relevant to an analysis [30] of quasielastic $(p,2p)$ scattering.

The fact that neglecting [27] the dependence of the absorption factor on the variable $\Delta\vec{r}_1$ is illegitimate was noticed in [28]. None the less the authors of ref. [28] did not accomplish a complete analysis of distortion effects in $A(e,e'p)$ scattering taking into account the dependence of the FSI factor on $\Delta\vec{r}_1$. They restricted themselves to accounting for the dependence of the FSI on the longitudinal component of the vector $\Delta\vec{r}_1$ and put in the FSI factor

$$\vec{b}_1 = \vec{b}_1^{\,\prime} = (\vec{b}_1 + \vec{b}_1^{\,\prime})/2\,. \tag{39}$$

For the reason that interaction between the two trajectories is a steep function of $\Delta\vec{b}_1 = (\vec{b}_1 - \vec{b}_1^{\,\prime})$, the approximation (39) can not be justified. Evidently, (39) precludes an accurate treatment of the transverse missing momentum distribution. Ref. [28] also used LDA for the one-body density matrix. It is easy to show using Eqs. (24), (32) that the approximation (39) leads to the $p_{m,z}$ integrated transverse missing momentum distribution which is proportional to the p_z integrated transverse SPMD. Furthermore, in the region $p_m \lesssim k_F$, where $n_f(p_m)$ may be approximated by the Gaussian form, in the resulting

three-dimensional missing momentum distribution the dependencies on the transverse and longitudinal components will factorize with having the same $p_{m\perp}$ dependence as the SPMD. Such a factorization can not be correct, because the term $\propto \Gamma^*\Gamma$ in Eq. (32), which has the most steep dependence on $\Delta\vec{b}_1$, and the terms $\propto \Gamma(\Gamma^*)$, which are smooth function of $\Delta\vec{b}_1$, have quite different dependence on $(z_1 - z_1^{'})$. Our numerical results show that three-dimensional missing momentum distribution (24) actually has a manifestly non-factorizable form. Therefore the application of the approximation (39) for the evaluation of the three-dimensional missing momentum distribution in $(e, e'p)$ scattering can not be justified.

Concluding discussion of the approaches of ref. [27, 28] one remark is in order on the LDA (38) for one-body density matrix that was used in [27, 28]. The LDA is widely believed to be a very good approximation for heavy nuclei. Our numerical results show that in the case of calculation the missing momentum distribution in $(e, e'p)$ scattering LDA turns out to be quite a crude approximation even for the nucleus mass number $A = 40$. The comparison of the results obtained using the full shell model density matrix (26) and its parameterization in a factorizable form (38) will be presented in section 6.

3 Connection between Glauber model and optical potential approach

A comparison between the Glauber formalism set up in section 2 and the optical potential approach (the conventional distorted-wave impulse approximation (DWIA)) that is usually used to describe the FSI effects in $(e, e'p)$ scattering at low Q^2 (for the recent review see ref. [18]), is in order. In the DWIA the FSI effects are taken into account by introducing a phenomenological optical potential, $V_{opt}(\vec{r})$. Then the distortion of the outgoing proton plane wave arises as a consequence of the eikonal phase factor

$$S_{opt}(\vec{r}) = \exp\left[-\frac{i}{v}\int\limits_{z}^{\infty} d\xi V_{opt}(\vec{b}, \xi)\right] \tag{40}$$

(v is the velocity of the struck proton). The missing momentum distribution in this approach is given by Eq. (24) with the following factored FSI factor

$$\Phi_{opt}(\vec{r}_1, \vec{r}_1^{\,'}) = S_{opt}(\vec{r}_1) S^*_{opt}(\vec{r}_1^{\,'}) \,, \tag{41}$$

The important feature of the DWIA is that optical potential does not depend on the individual coordinates of the spectator nucleons. Thus the optical potential in the DWIA embodies an effective description of the influence of the nuclear medium upon the wave function of the struck proton.

As a matter of fact, the Glauber model attenuation factor (3) is but the embodiment of solving of the wave equation for the wave function of the struck proton in the eikonal approximation as well. Nevertheless there is an important conceptual difference between the DWIA and the Glauber model approach. In the DWIA the FSI effects are taken into account at the level of the wave function of the ejected proton. However it is clear that a rigorous evaluation of the probability distribution for a subsystem (the struck proton in the case under consideration) for the process including a complex system (the system of the struck proton and spectator nucleons in our case) requires calculations at the level of the subsystem density matrix. Eqs. (3), (9) embody precisely this procedure in the case of quasielastic $(e, e'p)$ scattering. Indeed, as was mentioned above the Glauber model attenuation factor (3) realizes the solving of the wave equation for the outgoing proton wave function for a certain configuration of the spectator nucleons, $\tau = \{\vec{r}_2, ..., \vec{r}_A\}$. As usual we assume that at high energy of the struck proton one can neglect the motion of the spectator nucleons during the propagation of the fast struck proton through the residual nucleus. The averaging over the spectator nucleons positions of the reduced nuclear matrix element squared which was obtained through the wave equation relevant to the struck proton at fixed τ is performed in Eq. (9). It is clear that the averaging over τ is but the evaluation of the diagonal matrix element of the subsystem (the struck proton) in the momentum space. In another words the difference between the treatment of the FSI effects in the DWIA approach and in the Glauber model can be formulated as a difference in the order of operation. Schematically the order of operation in the DWIA

is as follows:

1. Averaging over the spectator nucleon positions (the evaluation of the effective optical potential).
2. Solving of the wave equation for the struck proton wave function using the effective optical potential and calculation of the reduced nuclear matrix element squared.

In the case of the Glauber model the reverse order is used:

1. Solving of the wave equation for the struck proton wave function at fixed positions of the spectator nucleons and computing of the reduced nuclear matrix element squared.
2. Averaging over the spectator nucleon positions.

In contrast to the optical potential FSI factor (41) the Glauber model factor (32) has a non-factorizable form due to the presence of the term $\propto \Gamma^*\Gamma$. The interaction between the two trajectories of the struck proton in the Glauber FSI factor (32) connected with $\Gamma^*\Gamma$ term is a consequence of the averaging over the spectator nucleon positions after computing the matrix element squared for fixed the spectator configuration τ. The physical origin of the $\Gamma^*\Gamma$ term in the FSI factor Φ (32) is the incoherent (see section 4) rescatterings of the struck proton on the spectator nucleons during its propagation through the target nucleus. Namely the sum over the nucleus excitations created by the elastic rescatterings of the ejected proton leads to the non-factorizable expression (32).

There is a formal analogy between the optical model FSI factor (41) and the Glauber model factor, if the $\Gamma^*\Gamma$ term in exponent (32) is excluded. Such a reduced attenuation factor, $\Phi^{Gl}_{opt}(\vec{r}_1, \vec{r}^{\,\prime}_1)$, takes on a factored form as a function of $\vec{r}_1$ and $\vec{r}^{\,\prime}_1$

$$\Phi^{Gl}_{opt}(\vec{r}_1, \vec{r}^{\,\prime}_1) = S^{Gl}_{opt}(\vec{r}_1) S^{Gl*}_{opt}(\vec{r}^{\,\prime}_1)\,, \tag{42}$$

with

$$S^{Gl}_{opt}(\vec{r}) = \exp\left[-\frac{1}{2}\sigma_{tot}(pN)(1 - i\alpha_{pN})t(\vec{b}, z)\right]\,. \tag{43}$$

The integral nuclear transparency (17) and the missing momentum distribution (24) calculated with using FSI factor (32) and (42) differ substantially. Our numerical results give clear cut evidence that the $\Gamma^*\Gamma$ term in (32) becomes very important in the region $|\vec{p}_m| \gtrsim 200$ MeV/c. This is a consequence of the short range (in the variable $(\vec{r}_1 - \vec{r}_1^{\,\prime})$) "interaction" between two trajectories in the FSI factor (32). It is clear that such a short range "interaction" can not be modeled in the optical potential approach even at the expense of any modification of the attenuation factor (41). Thus, the DWIA, which was successful in the region of low Q^2, can not be extended to description of inclusive $(e,e'p)$ reaction at high Q^2. The peculiarity of the high-Q^2 region is that in this case both the coherent rescattering of the struck proton on the spectator nucleons and the incoherent ones practically do not change the direction of the proton momentum. For this reason they need to be treated on the same footing. On the contrary, at low Q^2, when the energy of the struck proton is small, every incoherent rescattering of the struck proton leads to a considerable loss of the proton energy-momentum. As a result, the flux of the outgoing proton plane wave is suppressed. This effect is modeled in the DWIA by the imaginary part of the effective optical potential. Thus, the DWIA and the Glauber model appear to be applicable for description of $(e,e'p)$ scattering in different kinematical domains, at low Q^2 and at high Q^2, respectively.

It is interesting that at high Q^2 the optical potential form of the Glauber model FSI factor (42) still has a certain applicability domain. Namely, in a certain sense the attenuation factor (42) describes the FSI in exclusive $(e,e'p)$ scattering, when only the events with one knocked out nucleon (proton) are allowed. This connection between the FSI factor (42) and exclusive $(e,e'p)$ reaction takes place in the independent particle shell model. One can show that in the shell model without NN correlation the FSI factor (42) corresponds to the situation when the sum over the residual nucleus states includes only the states which arise from the ground state of the target nucleus after removing one of the protons. Indeed, in this case the index f in Eq. (2) indicates one of the states of the

target nucleus occupied by the protons. Thus Eq. (2) takes the form

$$M_f = \int d^3\vec{r}_1...d^3\vec{r}_A \Psi^*_{i,f}(\vec{r}_2,...,\vec{r}_A)\Psi_i(\vec{r}_1,...,\vec{r}_A)S(\vec{r}_1,...\vec{r}_A)\exp(i\vec{p}_m\vec{r}_1)\,, \tag{44}$$

here, as in Eq. (20), $\Psi_{i,f}$ is the $(A-1)$-body wave function of the system of $(A-1)$ nucleons obtained after removing of the proton in the state f from the target nucleus. Making use of the Slater determinant form of $\Psi_{i,f}$ and Ψ_i one can obtain to the leading order in $1/A$ for (44)

$$M_f = \frac{1}{\sqrt{Z}}\int d^3\vec{r}_1...d^3\vec{r}_A \exp(i\vec{p}_m\vec{r}_1)\phi_f(\vec{r}_1)\rho_f(\vec{r}_2,...,\vec{r}_A)S(\vec{r}_1,...\vec{r}_A)\,, \tag{45}$$

here ρ_f is $(A-1)$-body nuclear density defined by Eq. (20). As in section 2 we will use a factored representation (23) for this distribution. Then, making use of Eqs. (3,29) we arrive at the following expression for the matrix element (44)

$$M_f = \frac{1}{\sqrt{Z}}\int d^3\vec{r}_1 \exp(i\vec{p}_m\vec{r}_1)\phi_f(\vec{r}_1)S^{Gl}_{opt}(\vec{r}_1)\,. \tag{46}$$

Taking the sum of matrix elements (46) squared over the states f, immediately leads to the formula (24) with the reduced FSI factor (42) instead of the whole one (32). It is worth noting that from the quantum mechanical point of view, the FSI factor (42) describes the FSI effects from coherent rescatterings of the struck proton.

Making use of the FSI factors (32), (42) one can obtain for the integral nuclear transparency in quasielastic $(e,e'p)$ scattering

$$T^{inc}_A = \int d^2\vec{b}dz\rho_A(\vec{b},z)\exp[-\sigma_{in}(pN)t(\vec{b},z)]\,, \tag{47}$$

$(\sigma_{in}(pN) = \sigma_{tot}(pN) - \sigma_{el}(pN))$ for the inclusive reaction, and

$$T^{exc}_A = \int d^2\vec{b}dz\rho_A(\vec{b},z)\exp[-\sigma_{tot}(pN)t(\vec{b},z)]\,, \tag{48}$$

for the exclusive process.

Thus the integral nuclear transparency is controlled by the inelastic proton-nucleon cross section in the inclusive case and by the total proton-nucleon cross section in the

exclusive one. The physical origin of this difference is obvious. The incoherent rescatterings of the struck proton in the nuclear medium do not reduce the flux of the ejected proton. For this reason the attenuation is controlled by the inelastic proton-nucleon cross section in the inclusive case. In the exclusive $(e, e'p)$ reaction the processes with knocking out one (or more) of the spectator nucleons by the struck proton due to elastic incoherent rescatterings are forbidden. As a result, the attenuation is controlled by the total proton-nucleon cross section, and $T_A^{exc} < T_A^{inc}$.

As we already emphasized the correspondence between the FSI factor (42) and exclusive $(e, e'p)$ scattering takes place in the idealized shell model. Evidently, because of short range NN correlations, the above relationship between the missing momentum distribution calculated with the FSI factor (42) and the observed exclusive cross section will partly be lost. Indeed, $(e, e'p)$ scattering on the proton of the correlated NN pair leads to an ejection of the spectator nucleon of the correlated NN pair [21, 22], and the corresponding final state will not fall into the exclusive category. However, at small missing momenta $\vec{p}_m$ the probability of ejection of spectators must be small, because the typical momenta of nucleons in the correlated pair are of the same order in magnitude and $\gtrsim k_F$, and triggering on small $\vec{p}_m$ one effectively suppresses the contribution from correlated NN pairs.

4 Incoherent rescatterings and transverse missing momentum distribution

Eqs. (47), (48) show that in the case of the integral nuclear transparency a simple quasi-classical treatment of the FSI effects in $(e, e'p)$ scattering is possible to a certain extent. At the same time it is clear, that the missing momentum distribution (24) (or $T_A(\vec{p}_m)$ as a function of $\vec{p}_m$) does not admit a treatment at the classical level. The integration over $\vec{r}_1$ and $\vec{r}_1^{\,'}$ in Eq. (24) shows that experimentally observed cross section of $(e, e'p)$ scattering at some $\vec{p}_m$ is a result of manifestly quantum interference of amplitudes with

different positions where virtual photon strikes the proton. We would like to emphasize that in the case of the three-dimensional missing momentum distribution (24) the FSI effects from the incoherent rescatterings (connected with the term $\propto \Gamma^*\Gamma$ in the FSI factor (32)) taken separately also can not be treated at a classical level. Indeed, the low limit in the z-integration for the term $\propto \Gamma^*\Gamma$ in Eq. (32) is equal to the maximum value of the low limits of the z-integration in the terms $\propto \Gamma$ and $\propto \Gamma^*$. It implies that there is a part of the struck proton's trajectory where the incoherent rescatterings are forbidden. Due to this fact a probabilistic interpretation of the effects connected with the $\Gamma^*\Gamma$ term in the FSI factor (32) in the case of the nonintegrated distribution (24) is not possible.

In the case of the FSI factor (36) related to the $p_{m,z}$ integrated $\vec{p}_{m\perp}$ distribution (34), the low limits of z-integrations in the terms $\propto \Gamma(\Gamma^*)$ and $\propto \Gamma^*\Gamma$ are equal. It enables one to extend, to a certain extent, the probabilistic treatment, that is possible for the integral nuclear transparency (47), to the transverse missing momentum distribution (34). To demonstrate this fact it is convenient to rewrite (34) in the following form

$$\begin{aligned} w_\perp(\vec{p}_{m\perp}) = \frac{1}{(2\pi)^2}\int d^2\vec{b}dzd^2\vec{\Delta}\exp(i\vec{p}_{m\perp}\vec{\Delta})\rho(\vec{b}+\frac{1}{2}\vec{\Delta},z,\vec{b}-\frac{1}{2}\vec{\Delta},z) \\ \times S^{Gl}_{opt}(\vec{b}+\frac{1}{2}\vec{\Delta},z)S^{Gl*}_{opt}(\vec{b}-\frac{1}{2}\vec{\Delta},z)\exp\left[\eta(\vec{\Delta})\sigma_{el}(pN)t(\vec{b},z)\right]. \end{aligned} \tag{49}$$

Let us introduce a local momentum distribution including the distortion effects at the level of the optical form of the Glauber FSI factor (42) defined as follows

$$\begin{aligned} w_{\perp,opt}(\vec{b},z,\vec{p}_{m\perp}) = \frac{1}{(2\pi)^2}\int d^2\vec{\Delta}\exp(i\vec{p}_{m\perp}\vec{\Delta})\rho(\vec{b}+\frac{1}{2}\vec{\Delta},z,\vec{b}-\frac{1}{2}\vec{\Delta},z) \\ \times\rho_A^{-1}(\vec{b},z)S^{Gl}_{opt}(\vec{b}+\frac{1}{2}\vec{\Delta},z)S^{Gl*}_{opt}(\vec{b}-\frac{1}{2}\vec{\Delta},z). \end{aligned} \tag{50}$$

The local distribution (50) is normalized as

$$\int d^2\vec{p}_{m\perp}w_{\perp,opt}(\vec{b},z,\vec{p}_{m\perp}) = |S^{Gl}_{opt}(\vec{b},z)|^2.$$

Making use of the local missing momentum distribution (50) after expansion of the last exponential factor in Eq. (49) in a power-series, one can represent the formula (49)

in a form of ν-fold rescattering series

$$w_\perp(\vec{p}_{m\perp}) = \sum_{\nu=0}^{\infty} w_\perp^\nu(\vec{p}_{m\perp})\,, \tag{51}$$

where the zeroth order term is given by

$$w_\perp^0(\vec{p}_{m\perp}) = \int d^2\vec{b}dz\rho_A(\vec{b},z)w_{\perp,opt}(\vec{b},z,\vec{p}_{m\perp})\,, \tag{52}$$

and the contribution of ν-fold component for $\nu \geq 1$ reads

$$w_\perp^\nu(\vec{p}_{m\perp}) = \frac{1}{\nu!}\int d^2\vec{b}dz\rho_A(\vec{b},z)t^\nu(\vec{b},z)$$
$$\times \int \prod_{i=1}^{\nu} d^2\vec{q}_i \left(\frac{1}{\pi}\frac{d\sigma_{el}(pN)}{dq_i^2}\right) w_{\perp,opt}(\vec{b},z,\vec{p}_{m\perp} - \sum_{j=1}^{\nu}\vec{q}_j)\,. \tag{53}$$

Eqs. (51)-(53) embody the representation of the transverse missing momentum distribution in a form when all the quantum distortion effects are contained in the local missing momentum distribution computed with the FSI factor without $\Gamma^*\Gamma$ term. The contribution from the incoherent rescatterings connected with $\Gamma^*\Gamma$ term admits a probabilistic reinterpretation.

The transverse missing momentum distribution (34) obtained using the closure relation (8) is appropriate to inclusive $(e,e'p)$ scattering. Nevertheless the representation (51) can be used to estimate the contribution to the cross section of this process from the events with fixed number of the knocked out (recoil) nucleons. Our numerical results show that dominant contribution to the transverse missing momentum distribution in the region $p_\perp \lesssim k_F$ comes from the terms with $\nu \leq 1$. It means that inclusive cross section of $(e,e'p)$ scattering in the above kinematical domain is saturated by the events without and with one knocked out nucleon (besides the ejected proton). At high $p_{m\perp}$ the contribution from the terms with $\nu > 1$ becomes also important. The role of the incoherent rescatterings in the region $p_\perp \gtrsim k_F$ was recently discussed in [23]. It was shown, in particular, that incoherent rescatterings lead to a large tail in the missing energy distribution. The last exponent in Eq. (49) has the most steep dependence on the variable $\vec{\Delta}$. As a result, at high $p_\perp$ the incoherent rescattering effects become more important than the distortion

effects. In [23] only incoherent rescatterings were taken into account. As we will see in the region $p_\perp \lesssim k_F$ we are interested in the present paper, the relative magnitude of the FSI effects connected with distortion effects and incoherent rescatterings are of the same order and both of them must be taken into account simultaneously.

One remark on the physical interpretation of the incoherent rescatterings in quasielastic $(e, e'p)$ reaction, is in order. We would like to emphasize a somewhat formal nature of extension (51). It does not mean that the FSI effects connected with incoherent rescatterings allow the classical treatment. For instance, as we will see below, the representation (51) even does not imply that the momentum transfers in the incoherent rescatterings of the struck proton on spectator nucleons are purely transverse. For this reason, in particular, it is not possible to model the FSI effects associated with the incoherent rescatterings of the struck proton in the nuclear medium by virtue of the Monte-Carlo approach.

5 Longitudinal missing momentum distribution and applicability limits of Glauber model

The failure of the quasiclassical probabilistic treatment of the incoherent rescatterings becomes especially evident in the case of the longitudinal missing momentum distribution. Indeed, naively, from the classical point of view, one can expect that this distribution is not affected by the elastic rescatterings of the struck proton on the spectator nucleons. In fact, as one can see from Eqs. (24), (32), because of the last term in the exponent in Eq. (32), the incoherent rescatterings must affect, and have a quite nontrivial impact on, the longitudinal missing momentum distribution.

It is worth noting that even from a simple qualitative quantum mechanical consideration one can understand that the incoherent rescattering must influence upon the longitudinal missing momentum distribution. Indeed, let Δl be the distance between the point where the virtual photon strikes the ejected proton and the point where incoherent scattering off the spectator nucleon takes place. Then, it is evident from the uncertainty

relation that the momentum transfer in the pN scattering has uncertainty $\Delta k \sim 1/\Delta l$. In the case of sufficiently small Δl the longitudinal momentum transfer can be comparable to the transverse one. Indeed it is evident from the uncertainty relation that in the situation when Δl is about the proton size both longitudinal and transverse momentum transfer will be about the inverse proton radius. Thus, one can conclude that in the region of high $|p_{m,z}|$, $p_{m\perp}$ ($|p_{m,z}| \sim p_{m\perp}$) the incoherent rescatterings of the struck proton on the adjacent spectator nucleons must considerably affect the missing momentum distribution as compared to the PWIA case. As we will see the numerical results actually show that the additional $\Gamma^*\Gamma$ term in the FSI factors (32), (35) gives rise a considerable tail at high $|p_{m,z}|$, which is missed if the optical potential form (42) of the FSI factor is used.

Formally, the sensitivity of the longitudinal missing momentum distribution to the incoherent rescatterings is connected with the above mentioned peculiarity in the z-integration for the term $\propto \Gamma^*\Gamma$ in Eq. (32). Indeed, the function $\Phi^{Gl}_{opt}(\vec{r}_1, \vec{r}_1^{\,'})$ defined by Eq. (42) has a smooth behavior in the variable $\xi = z_1 - z_1^{'}$ at the point $\xi = 0$. On the contrary, the FSI factor (32) as a function of ξ has a discontinuous derivative with respect to ξ at the point $\xi = 0$. The origin of this discontinuity is the appearance in the $\Gamma^*\Gamma$ term of the nonanalytical function $max(z_1, z_1^{'})$ as the low limit in the z-integration. Evidently, the singular behavior of the integrand in Eq. (24) (and Eq. (33) as well) in the variable ξ, after Fourier transform will show itself up as an anomalous behavior of the missing momentum distribution at high longitudinal momenta.

Let us proceed with the analysis of the situation with the longitudinal momentum distribution for the case of the $\vec{p}_{m\perp}$ integrated distribution (33). The absence of the integration over $(\vec{b} - \vec{b}^{\,'})$ makes this case considerably more simple, as compared to the nonintegrated one (24), for a qualitative analysis.

It is convenient to rewrite the longitudinal FSI factor (35) in such a form

$$\Phi_z(\vec{b}, z, z^{'}) = \Phi^{in}_{z,opt}(\vec{b}, z, z^{'}) C_1(\vec{b}, z, z^{'}) C_2(\vec{b}, z, z^{'})\,, \qquad (54)$$

with

$$\Phi^{in}_{z,opt}(\vec{b},z,z^{'}) = \exp\left[-\frac{1}{2}\sigma_{in}(pN)t(b,z) - \frac{1}{2}\sigma_{in}(pN)t(b,z^{'})\,,\right] \tag{55}$$

$$C_1(\vec{b},z,z^{'}) = \exp\left[\frac{i}{2}\sigma_{tot}(pN)\alpha_{pN}\left(t(b,z)-t(b,z^{'})\right)\right]\,, \tag{56}$$

$$C_2(\vec{b},z,z^{'}) = \exp\left[-\frac{1}{2}\sigma_{el}(pN)\left|t(b,z)-t(b,z^{'})\right|\right]\,. \tag{57}$$

In Eq. (54) we singled out from the FSI factor (33) the function $\Phi^{in}_{z,opt}(\vec{b},z,z^{'})$. In a certain sense it can be interpreted as the optical potential FSI factor taking into account only the distortion of the plane wave connected with the real inelastic interactions of the struck proton in the nuclear medium. We will refer to the corresponding missing momentum distribution as $w^{in}_{z,opt}(p_{m,z})$. $\Phi^{in}_{z,opt}(\vec{b},z,z^{'})$ is a symmetric function of z, $z^{'}$. For this reason $w^{in}_{z,opt}(p_{m,z})$ is an even function of $p_{m,z}$.

For the purpose of the qualitative analysis in the case of $A \gg 1$ one can approximate the functions $C_{1,2}(\vec{b},z,z^{'})$ by the following expressions

$$C_1(\vec{b},z,z^{'}) = \exp\left[-ik_1(z-z^{'})\right]\,, \tag{58}$$

$$C_2(\vec{b},z,z^{'}) = \exp\left[-k_2|z-z^{'}|\right]\,, \tag{59}$$

where

$$k_1 = \frac{1}{2}\sigma_{tot}(pN)\alpha_{pN}\langle n_A\rangle\,, \tag{60}$$

$$k_2 = \frac{1}{2}\sigma_{el}(pN)\langle n_A\rangle\,, \tag{61}$$

and $\langle n_A\rangle$ is the average nuclear density.

Then, using these approximations, the longitudinal missing momentum distribution may be represented in such a convolution form

$$w_z(p_{m,z}) \approx \frac{1}{2\pi}\int dk w^{in}_{z,opt}(p_{m,z}-k_1-k)c_2(k)\,. \tag{62}$$

Here we used notation $c_2(k)$ for the Fourier transform of the factor C_2 approximated by formula (59). It is clear from Eq. (62) that the major effect of the nonzero real part of the pN-amplitude contained in the factor C_1 is a shift of the longitudinal missing momentum

distribution by k_1 [31]. In the region $Q^2 \sim 2-10$ GeV^2 the shift is quite large $k_1 \sim 20$ MeV/c. Thus nonzero α_{pN} leads to asymmetry of $p_{m,z}$ distribution about $p_{m,z}=0$. The role of the factor C_2 is more interesting. The Fourier transform of the factor C_2 for the case of the approximation (59) is given by

$$c_2(k) = \int d\xi \exp(ik\xi)\exp(-k_2|\xi|) = \frac{2k_2}{k^2+k_2^2}\,. \tag{63}$$

In the kinematical domain we are interested in $k_2 \sim 10-20$ MeV/c, and the inequality $k_2 \ll k_F$ is satisfied. It means that the Fourier transform of C_2 when approximation (59) is used has a form of a sharp peak with width that is much less then the width of the Fermi distribution. In the real situation when exact expression (57) is used the width of the peak of the Fourier transform of the function $C_2(\vec{b},z,z^{'})$ in the variable $\xi=(z-z^{'})$ will be controlled by the inverse nucleus size because $k_2 \lesssim 1/R_A$ for real nucleus. Due to inequality $k_F \gg 1/R_A$ this width again turns out to be much less than the width of the Fermi distribution. The finite nucleus size will not change the asymptotic law $c_2(k) \propto k^{-2}$ connected only with a nonanalytical behavior of expressions (57), (59) at the point $z=z^{'}$. From the above consideration it is clear that at $|p_{m,z}| \ll k_F$ the factor $c_2(k)$ in the convolution representation (62) acts like δ-function. Hence, at small $|p_{m,z}|$ we will have

$$w_z(p_{m,z}) \approx w_{z,opt}^{in}(p_{m,z}-k_1)\,. \tag{64}$$

However, at sufficiently large $|p_{m,z}| \gtrsim k_F$ the $p_{m,z}$ dependence of $w_z(p_{m,z})$ will be controlled by the asymptotic behavior of the factor $c_2(k)$, and the following regime will set in

$$w_z(p_{m,z}) \propto p_{m,z}^{-2}\,. \tag{65}$$

Our numerical results show that already in the region $|p_{m,z}| \sim 200-300$ Mev/c there is a considerable deviation from the approximate formula (64). The missing momenta at which the onset of the regime (65) takes place are considerably greater than k_F. The independent particle shell model used in the present paper is not applicable for analysis of $(e,e'p)$ scattering in this kinematical domain.

Let us consider in detail the physical reason for the appearance of the longitudinal momentum transfer in the incoherent rescatterings of the struck proton in $(e,e'p)$ reaction. The origin of the longitudinal momentum transfer in $(e,e'p)$ scattering is connected with the absence in this case of an incoming proton plane wave. Indeed, as it was mentioned above the missing momentum distribution (24) from the quantum mechanical point of view corresponds to the interference of the amplitudes with different positions at which the virtual photon strikes the proton. For each of this amplitudes the wave function of the spectator nucleons, after the struck proton leaves the target nucleus, will be distorted along the straight line that begins from the point where the photon-proton interaction takes place. It is clear that decomposition of the distortion of the spectator nucleon wave function into the plane waves contains besides the components with transverse momentum the components with longitudinal momentum. The asymptotic behavior (65) of the longitudinal momentum distribution is a consequence of the discontinuous distortion of the spectator nucleon wave functions. This discontinuity is connected with θ-function appearing in the Glauber model attenuation factor (3). It is evident that allowance for the finite longitudinal size, d_{int}, of the region where proton-nucleon interaction takes place, must lead to a smearing of the sharp edge of the θ-function in Eq. (3). Evidently, d_{int} is about the proton radius. As a consequence of the smearing of θ-function in Eq. (3) the $p_{m,z}^{-2}$ law (65) will be replaced by a somewhat steeper decrease at high $|p_{m,z}|$. We can suggest as a generalization of Eq. (3) to the case of finite d_{int} the same equation in which one of the following replacements is made

$$\theta(z) \Rightarrow \frac{\sqrt{\pi}}{d_{int}} \int\limits_{z}^{\infty} d\xi \exp(-\xi^2/d_{int}^2)\,, \tag{66}$$

or

$$\theta(z) \Rightarrow \frac{1}{2}\left[1+\tanh(z/d_{int})\right]\,. \tag{67}$$

Of course, there are no serious theoretical motivations to use either of the prescriptions (66) or (67) at such $|p_{m,z}|$, where the missing momentum distributions corresponding

to the attenuation factor (3) and obtained with using replacement (66) or (67) differ strongly. Nevertheless, they can be used to clarify the applicability limits of the standard Glauber model. Fortunately, it turns out that the kinematical range of the longitudinal missing momentum, where the Glauber model is still applicable, is quite broad. In [13, 14] it was checked that θ-function Ansatz in the Glauber attenuation factor in the case of deuteron works very well in the region $|p_{m,z}| \lesssim 500$ MeV/c. Our numerical calculations in the kinematical domain $|p_{m,z}| \lesssim 300$ MeV/c also show that introduction of the finite interaction size $d_{int} \sim 1\ fm$ practically does not change the standard Glauber model predictions obtained with using θ-function in Eq. (3).

In connection with above discussion it is worth noting the following. In our analysis we neglected the short range NN correlations. Due to NN repulsive core the probability to find in the target nucleus two nucleons at the same point really is suppressed. It means that at high $|p_{m,z}|$ even without allowance for finite d_{int} the tail longitudinal missing momentum distribution will decrease considerably steeper when the short range NN correlation are taken into account. Since NN correlation radius $\sim 1\ fm$ this effect can be neglected in the region $|p_{m,z}| \lesssim 300$ MeV/c likewise one from the finite d_{int}.

The found incompleteness of the Glauber model in the case of $(e, e'p)$ scattering in high-$|p_{m,z}|$ region makes questionable the possibility of using the measured missing momentum distribution for obtaining the information on the short range NN correlations in nuclei. It is clear now that besides the short range NN correlations, the missing momentum distribution at high $|p_{m,z}|$ probes the nucleon structure as well. Notice, that the sensitivity of high-$|p_{m,z}|$ tails to the nucleon size survives at high Q^2 as well. For this reason we will face the same problem in the analysis of $(e, e'p)$ scattering at high Q^2, in the CT regime of large contribution from the off-diagonal inelastic rescatterings.

In conclusions of this section we would like to emphasize that in the case of reaction $(e, e'p)$ we face a situation which is quite different from the small angle hadron-nucleus scattering. In the latter case the incoming hadron plane wave exists. Hence, the θ-function effect in the Glauber model attenuation factor (3) disappears. As a result, the Glauber

model predictions do not contain uncertainties connected with the finite longitudinal size d_{int}. However, it is clear that this problem will arise in the case of quasielastic $(p,2p)$ scattering at large angle. The analysis of the reaction $(p,2p)$ in the kinematical range of the BNL experiment [32] will be presented elsewhere.

6 Numerical results

In this section we present our numerical results for the missing momentum distribution and nuclear transparency in quasielastic $(e,e'p)$ scattering obtained in the Glauber formalism. To avoid complications with the target nucleus spin we restricted ourselves to the case of closed shell nuclei ^{16}O and ^{40}Ca. We adjusted the oscillator shell model frequency, ω_{osc}, for these two nuclei to reproduce the experimental value of the root-mean-square radius of the charge distribution, $\langle r^2\rangle^{1/2}$. We used the values [33] $\langle r^2\rangle^{1/2} = 2.73\ fm$ for ^{16}O, and $\langle r^2\rangle^{1/2} = 3.47\ fm$ for ^{40}Ca, which correspond to the oscillator radius, $r_{osc} = (m_p\omega_{osc})^{-1/2}$, equal to 1.74 fm for ^{16}O and 1.95 fm for ^{40}Ca. The difference between the charge distribution and the proton nuclear density connected with the proton charge radius was taken into account.

As it was stated in section 2 we use the exponential parameterization of the proton-nucleon elastic amplitude. The diffraction slope of the pN scattering was estimated from the relation

$$B_{pN} \approx \frac{\sigma^2_{tot}(pN)(1+\alpha^2_{pN})}{16\pi\sigma_{el}(pN)}\,. \tag{68}$$

In our calculations we define the pN cross sections and α_{pN} as mean values of these quantities for pp and pn scattering. We borrowed the experimental data on pp, pn cross sections and α_{pp}, α_{pn} from the recent review [34]. From the point of view of the Q^2 dependence of the Glauber model predictions for the missing momentum distribution in $(e,e'p)$ scattering in the region $Q^2 \sim 2-10$ GeV^2, especially important is the energy dependence of B_{pN} and $\sigma_{el}(pN)$. We remind that the typical kinetic energy of the struck proton $T_{kin} \approx Q^2/2m_p$. Typically B_{pN} rises from $B_{pN} \approx 4.5$ GeV^{-2} at $Q^2 = 2$ GeV^2 to

$B_{pN} \approx 8$ GeV^{-2} at $Q^2 = 10$ GeV^2, and $\sigma_{el}(pN)$ falls from $\sigma_{el}(pN) \approx 23$ mb at $Q^2 = 2$ GeV^2 to $\sigma_{el}(pN) \approx 11.5$ mb at $Q^2 = 10$ GeV^2. In our kinematical domain $\sigma_{tot}(pN)$ slightly decreases with Q^2, $\sigma_{tot}(pN) \approx 43.5\, mb$ at $Q^2 = 2$ GeV^2 and $\sigma_{tot}(pN) \approx 40\, mb$ at $Q^2 = 10$ GeV^2. We remind that in the Glauber model the trajectories of the high energy particles are assumed to be straight lines. In our case the struck proton momentum ~ 2 GeV at $Q^2 \sim 2$ GeV^2. In this region of momenta the mean value of the momentum transfer in pN scattering is $\sim 1/\sqrt{B_{pN}} \sim 0.45$ GeV/c. Thus there are reasons to believe that the Glauber formalism is still reliable at lower bound of the kinematical domain we are interested in the present paper $Q^2 \sim 2 - 10$ GeV ^{2}.

To illustrate the role of the incoherent rescatterings in process $(e, e'p)$ we present a systematical comparison of results obtained for the full Glauber theory with $\Gamma^*\Gamma$ term in the FSI factor (32) included, and the truncated version when $\Gamma^*\Gamma$ term not included. We remind that from the point of view of the independent particle shell model these two versions are relevant to the inclusive and exclusive conditions in $(e, e'p)$ scattering, respectively.

The results for the integral nuclear transparencies T_A^{inc} and T_A^{exc} defined by Eqs. (47), (48) are shown in Fig. 1. Because of the rise of $\sigma_{in}(pN)$, T_A^{inc} slowly decreases in our kinematical range. T_A^{exc}, which is controlled by $\sigma_{tot}(pN)$, is approximately flat. As one can see from Fig. 1 the replacement of $\sigma_{in}(pN)$ by $\sigma_{tot}(pN)$ considerably reduces the integral nuclear transparency. We use the computed values of T_A^{inc} and T_A^{exc} to obtain normalized to unity missing momentum distribution (18) for the cases with $\Gamma^*\Gamma$ term in the FSI factor (inclusive $(e, e'p)$ scattering) and without the one (exclusive $(e, e'p)$ scattering).

The integral nuclear transparencies for the case when the kinematical domain D in the definition (1) includes all missing momenta, depends only on the diagonal component of the one-body nuclear density matrix , *i.e.*, the nuclear density. On the contrary, the missing momentum distribution and $T_A(\vec{p}_m)$ as a functions of $\vec{p}_m$ or the nuclear transparency for a certain kinematical region D , $T_A(D)$, defined by Eq. (1) are controlled by the whole one-body density matrix. The major part of the results presented in this

section have been obtained using oscillator shell model density matrix (26). In order to check the sensitivity of the results to the form of the one-body density matrix, we also performed in a few cases the calculations using the LDA parameterization (38) of the one-body density matrix. A comparison of the results obtained in these two versions is very interesting from the point of view of clarifying the accuracy and applicability limits of the LDA parameterization (38) which is widely used in the literature for nuclei with large nuclear mass number A.

In Fig. 2, 3 we show the angular dependence of the ratio of the normalized missing momentum distribution $n_{eff}(p_m,\theta)$ to the Fermi momentum distribution $n_F(p_m)$ for p_m =150, 200, 250 and 300 MeV/c at Q^2 =2 and 10 GeV^2 . The forward-backward asymmetry of this ratio is a consequence of the nonzero real part of the elastic pN amplitude. The appearance of a bump for p_m = 300 MeV/c at $\theta \approx 100^o$ for Q^2 = 2 GeV^2 in the version with $\Gamma^*\Gamma$ term is connected with the fact that momentum transfers in the incoherent rescatterings are predominantly transverse. At Q^2 = 10 GeV^2, the bump evolved into the shoulder, which is related to the higher value of the diffraction slope. In contrast to the version with $\Gamma^*\Gamma$ term, in the case without $\Gamma^*\Gamma$ term we obtained a dip at $\theta \sim 80^o$ for $p_m \sim 250-300$ MeV/c. Thus we see that quantum FSI effects related to the elastic rescatterings of the struck proton without the excitation of the residual nucleus lead to a considerable distortion of the outgoing proton plane wave.

Fig. 4 illustrates p_m dependence of the nonintegrated nuclear transparence $T_A(p_m,\theta)$ for $\theta = 0^o$, 90^o, 180^o at Q^2 = 2 GeV^2. The results presented in Fig. 4 more clearly demonstrate the relative role played by the absorption effects connected with the terms $\propto \Gamma(\Gamma^*)$ and the incoherent rescatterings effects related to the $\Gamma^*\Gamma$ term in the full FSI factor. We see that absorption leads to appearance of a deep dip in the nuclear transparency in $(e,e'p)$ reaction for exclusive conditions at $p_m \sim 270-300$ MeV/c in the case of transverse kinematics ($\theta = 90^o$). With allowance for the incoherent rescatterings ($\Gamma^*\Gamma$ term is included) the nuclear transparency in transverse kinematics steeply rises at $p_m \gtrsim 250$ Mev/c. Even in the parallel kinematics ($\theta = 0^o$ and $\theta = 180^o$) the effect of

$\Gamma^*\Gamma$ term becomes significant at $p_m \gtrsim 250$ Mev/c. This effect is a manifestation of the longitudinal momentum transfer discussed in section 5. Thus the results presented in Fig. 4 show that the contribution from the incoherent rescatterings becomes important only at sufficiently large missing momenta, $|\vec{p}_m| \gtrsim 200 - 250$ MeV/c.

Notice, that Fig. 4 already demonstrates that the three-dimensional missing momentum distribution has a substantially non-factorizable dependence on the transverse and longitudinal components of the missing momentum. In order to demonstrate the degree of violation of the $p_{m\perp}$–$p_{m,z}$ factorization we present in Fig. 5 the nuclear transparency versus $p_{m\perp}$ at different $p_{m,z}$ for ^{40}Ca at $Q^2 = 2$ GeV^2. As one can see the $p_{m\perp}$–$p_{m,z}$ factorization is manifestly violated. Both versions, with $\Gamma^*\Gamma$ term and without one, demonstrate that $p_{m\perp}$ dependence of $T_A(\vec{p}_\perp, p_{m,z})$ is strikingly different from that of the Fermi momentum distribution. It makes it clear, in particular, that approach proposed in ref.[28] can not be justified. As it was explained in section 2, the approximation (39) of ref. [28] leads to the almost $p_{m\perp} - p_{m,z}$ factorizable form of $w(\vec{p}_m)$ with the same $p_{m\perp}$ dependence as for the Fermi momentum distribution.

In Fig. 6, 7 we show the behavior of the nuclear transparency versus $p_{m\perp}$ in the case when the kinematical domain D in the definition (1) includes all the longitudinal momenta. From the point of view of the representation of $w_\perp(\vec{p}_{m\perp})$ by the multiple scattering series (51), the version with $\Gamma^*\Gamma$ term kept corresponds to the situation when all incoherent rescatterings are taken into account. The case without $\Gamma^*\Gamma$ term in the FSI factor is equivalent to keeping only the zeroth order term in series (51). Besides these two cases we present in Fig. 6, 7 the results for the case when two first terms in the series (51) are included, $\nu = 0$ and $\nu = 1$. As one can see from Fig. 6, 7 in the kinematical domain under consideration the mechanisms with $\nu = 0,\ 1$ practically saturate the cross section of inclusive $(e, e'p)$ reaction. The incoherent rescatterings of the struck proton become important at $p_{m\perp} \gtrsim 200$ MeV/c.

The $p_{m,z}$ dependence of the nuclear transparency for the case when the events with all transverse missing momentum are included are shown in Fig. 8, the solid and dashed

curves are for the version with and without $\Gamma^*\Gamma$ term. As one can see, with increasing of nuclear mass number, the development of the clear cut two-dip structure takes place for the case without $\Gamma^*\Gamma$ term. Thus the attenuation effects connected with the FSI of the struck proton considerably affect the missing momentum distribution as compared to the PWIA case. Fig. 8 shows that the including of the $\Gamma^*\Gamma$ term leads to the appearance of large tails in the longitudinal missing momentum distribution at $|p_{m,z}| \gtrsim 250$ MeV/c. As in the case of a purely parallel kinematics (see below), there is a considerable asymmetry about $p_{m,z} = 0$ connected with nonzero α_{pN}. The integral asymmetry, A_z, defined as

$$A_z = \frac{N(p_{m,z} > 0) - N(p_{m,z} < 0)}{N(p_{m,z} > 0) + N(p_{m,z} < 0)} \tag{69}$$

($N(p_{m,z} > 0)$ and $N(p_{m,z} < 0)$ are the number of events with $p_{m,z} > 0$ and $p_{m,z} < 0$, respectively), is large, $A_z \approx -(0.07 - 0.08)$, and approximately constant, in our kinematical region ($Q^2 \sim 2 - 10$ GeV^2).

Notice that the large $p_{m,z}$ asymmetry, obtained in the Glauber model, obscures the study of the CT effects in quasielastic $(e, e'p)$ scattering at $Q^2 \lesssim 10$ GeV^2 using the experimental data on the dependence of the nuclear transparency on the Bjorken x (remind that $x \approx (1 + p_{m,z}/m_p))$ [10, 11, 12] . The point is that at $Q^2 \sim 5 - 10$ GeV^2 the expected values of the asymmetry because of the contribution of the inelastic (off-diagonal) scattering of the struck proton on spectator nucleons [11, 12] turn out to be by factor 2-4 smaller than the above Glauber theory prediction for A_z. The nonzero α_{pN} leading to the $p_{m,z}$ asymmetry in the Glauber model predictions is connected with the contributions to the elastic pN amplitude from the secondary reggeons. As it is known [35] the reggeon exchange requires a finite formation time increasing with hadron energy. Thus one can expect that due to this effect real part of the effective elastic pN amplitude in the nuclear medium in the case of $(e, e'p)$ scattering will partially differ from the one measured in pN scattering in vacuum. In the absence of a rigorous theoretical model for the reggeon exchanges it will be difficult to disentangle the CT contribution to the $p_{m,z}$ asymmetry. Nevertheless any rise of A_z with Q^2 will signal the onset of the CT effects, since the finite formation time effects for the reggeon exchange can only reduce A_z predicted in the

standard Glauber model.

It is instructive to compare the $p_{m,z}$ dependence of the nuclear transparency for the case of the parallel kinematics , $p_{m\perp} = 0, \theta = 0^o,\ 180^o$ shown in Fig. 4 with the $p_{m,z}$ dependence when all $\vec{p}_{m\perp}$ are included, Fig. 8. Such a comparison very clearly demonstrates, that the major contribution from incoherent rescatterings into large-$|p_{m,z}|$ tails comes from the region of the sufficiently large $p_{m\perp}$. The same conclusion can be made from the curves presented in Fig. 5. We remind that namely this pattern of the contribution to the missing momentum distribution from the incoherent rescatterings of the struck proton on the adjacent spectator nucleons at high-$|p_{m,z}|$ region was predicted in section 5 on the basis of the uncertainty relation. Thus, our numerical results give clear cut evidence in favor of necessity to treat the incoherent rescatterings of the struck proton in quasielastic $(e, e'p)$ reaction in a quantum mechanical manner.

The above discussed peculiarities of the contribution of the incoherent rescatterings into three-dimensional missing momentum distribution in $(e, e'p)$ scattering are very important from the point of view a comparison of the theoretical predictions with experimental data on the nuclear transparency obtained for a certain kinematical domain D. Indeed, the integral nuclear transparencies T_A^{inc} and T_A^{exc} corresponding the calculations with and without $\Gamma^*\Gamma$ term in the FSI factor differ substantially. The difference is connected with allowance for the incoherent rescatterings in the case of inclusive $(e, e'p)$ reaction. However as we see from our numerical results the contribution from the incoherent rescatterings become important only at sufficiently large missing momenta. It means that the nuclear transparency measured in a certain restricted kinematical region D may considerably differ from the integral nuclear transparency even if in both cases the inclusive experimental conditions are imposed. Of course the fact that the contribution from the incoherent rescatterings predominantly comes from high- $|\vec{p}_m|$ region does not mean automatically that the nuclear transparency measured in the kinematical region including sufficiently small $|\vec{p}_m|$ will be close to T_A^{exc}. As we have seen even if $\Gamma^*\Gamma$ term is not taken into account the missing momentum distribution will considerably differ from

the PWIA case.

The above is especially important for experimentally disentangling CT effects in $(e, e'p)$ scattering. It is clear that definitive conclusions on their role can only be made if one compares the experimental nuclear transparency with the one obtained in the Glauber model for the same kinematical domain D. In order to demonstrate the dependence of the nuclear transparency on the choice of kinematical domain D in the definition (1) in Fig. 9, 10 we present the results in the case of the version with $\Gamma^*\Gamma$ term (solid lines) for four different windows in the missing momentum. For the purpose of the comparison we also show in these figures the inclusive and exclusive integral nuclear transparency computed using $\sigma_{in}(pN)$ (long-dashed lines) and $\sigma_{tot}(pN)$ (dash-dotted lines). To illustrate the dependence of the Glauber model predictions on the parameterization of the one-body density matrix, we show in Fig. 9, 10 the results obtained using the LDA (38) for the one-body density matrix (short-dashed line). The results for shell model one-body density matrix and its LDA parameterization differ substantially, and this difference varies with the missing momentum window. As one can see from Fig. 9, 10 the Glauber theory results with full shell model density matrix for considered kinematical windows do considerably differ from the both integral nuclear transparencies T_A^{inc} and T_A^{exc}. Important finding is that despite the increase of $\sigma_{in}(pN)$ for all considered kinematical domains the nuclear transparency slightly rises with Q^2. We wish specially emphasize sensitivity of $T_A(D)$ to the missing momentum window. One could expect that intranuclear attenuation can neither be stronger than given by σ_{tot} nor weaker than given by σ_{in}, hence naively

$$T_A^{inc} > T_A(D) > T_A^{tot}$$

The results shown in Fig. 9, 10 clearly demonstrate, that quantum mechanical distortion effects do not amount to a naive attenuation. Namely, even at small $\vec{p}_m$, where incoherent elastic rescattering effects are still small one can easily find $T_A(D) > T_A^{inc}$.

The kinematical region $D = (p_{m\perp} < 250,\ |p_{m,z}| < 50$ MeV/c) approximately corresponds to the kinematical conditions of the recent NE18 experiment [8]. In Fig. 11 we compare the experimental data [8] for ^{12}C and ^{56}Fe with the Glauber model predictions.

^{12}C and ^{56}Fe are not closed-shell nuclei, $T_A(D)$ for these nuclei were calculated assuming that they interpolate between T_A^{exc} and T_A^{inc} as for the closed-shell ^{16}O and ^{40}Ca nuclei, respectively. T_A^{inc} and T_A^{exc} for ^{12}C were computed using the parameterization of the nuclear density in a form of a sum of Gaussians [33]. In the case of ^{56}Fe the three-parameter Gaussian model [33] was used. The difference between the charge density distribution and the proton density distribution was taken into account. Strong dependence of $T_A(D)$ on the missing momentum window D, see Fig. 9, 10, makes the full quantum mechanical treatment of distortions imperative for a quantitative comparison between the theory and experiment. This effect is missed in all the previous calculations of T_A, which were reviewed to much detail by Makins and Milner [36]. The only exception is paper [31], which discussed how $T_A(D_{NE18})$ interpolates between T_A^{exc} and T_A^{inc}, but the analysis [31] only included the dominant distortion of the transverse momentum distribution from the incoherent rescatterings. In Fig. 11 we also show the estimate [7] for CT effects.

Our values of $T_A(D_{NE18})$ are somewhat below the NE18 determinations. To this end, we wish to remind that the NE18 analysis [8] uses certain model evaluations of the denominator in Eq. (1). One of the key assumptions of is that, modulo to the overall normalization, the nuclear spectral function is identical to the PWIA spectral function. Our results show, that this can not be correct for the FSI effects, but the accuracy of the NE18 experiment is not sufficiently high as to unravel the distortions of the size found in our analysis. Furthermore, the NE18 analysis introduces renormalization of T_A by the factor 1.11 ± 0.03 for ^{12}C and 1.22 ± 0.06 for ^{56}Fe nuclei, which renormalization is meant to account for the missing strength associated with the large-p_m component of the spectral function coming from short range NN correlations. Similar correction for the missing strength must be included, both in the numerator and denominator of Eq. (1), in our analysis too. The recent work [13] found strong interference between, and similar strength of, the FSI and short range correlation effects, which make the corrections for the NN correlations to the numerator and denominator different. For this reason the above cited renormalization effects must be regarded as an indication of the accuracy of

shell model calculations of $T_A(D_{NE18})$. Calculations with NN correlations and correct treatment of FSI are required for the higher accuracy comparison of the experimental and theoretical values of T_A.

The difference between the theoretical predictions obtained using the full shell model one-body density matrix and the density matrix of the LDA (38) shown in Fig. 9, 10 shows that in the case of three-dimensional missing momentum distribution the FSI effects in quasielastic $(e, e'p)$ scattering is rather sensitive to the non-diagonal elements of the one-body density matrix. To gain more insight into the sensitivity to the one-body density matrix, in Fig. 12 we compare the results for the $p_{m,z}$ integrated nuclear transparency as a function of $p_\perp$ for the full shell model density matrix and the LDA density matrix (the solid and long-dashed curves) at $Q^2 = 2$ GeV^2. The difference of $p_{m,z}$ integrated transparencies is very large at small $p_{m\perp}$, reaching ~ 20 % for ^{40}Ca nucleus, the full shell model also predicts much deeper minimum of transparency at $p_{m\perp} \sim 225$ MeV/c, beyond the crossover at $p_{m\perp} \sim 150$ MeV/c.

The difference between the Glauber model predictions for two versions of the one-body density matrix become more clear if one compares the results in the case of the non-integrated nuclear transparency. In Fig. 13, 14 we perform this comparison for the transverse kinematics. The results for the parallel kinematics presented in Fig. 15, 16. As one can see from Fig. 13-16, using the LDA (38) leads to an underestimation of the nuclear transparency at small $|\vec{p}_m|$ and to an overestimation in high-$|\vec{p}_m|$ region. At $\vec{p}_m \sim 0$ predictions from full shell model and from LDA differ by $\gtrsim 20\%$.

One could expect that the LDA (38) will become more reliable with increasing of A. Our important finding is that the discrepancy between the Glauber model predictions for two versions of the one-body density matrix does not reveal any tendency to disappear with increasing of the nucleus mass number. This fact once more emphasizes that the quantum interference effects play important role in the FSI of the struck proton in the nuclear medium. The failure of the LDA even for sufficiently heavy nucleus ^{40}Ca at the qualitative level can be explained by the large contribution to the cross section of $(e, e'p)$

scattering from the events corresponding to the ejecting of the proton from the nucleus surface. Evidently there are no any reasons to expect that the LDA will be reliable in the surface region.

7 Conclusions

The purpose of this work has been a study of the missing momentum distribution in quasielastic $(e,e'p)$ scattering in the region of moderate missing momenta , $|\vec{p}_m| \lesssim 300$ MeV/c, and high energy, $Q^2 \sim 2-10$ GeV^2, within the Glauber multiple scattering theory. To perform such an analysis, we generalized the Glauber theory developed for the hadron-nucleus collisions at high energy, to the case of $(e,e'p)$ reaction. We presented for the first time a consistent treatment of the novel effect of interaction between the two trajectories which enter the calculation of the FSI-modified one body density matrix and have an origin in the incoherent elastic rescatterings of the struck proton.

Our numerical results show that the missing momentum distribution in $(e,e'p)$ scattering are substantially affected by FSI effects as compared to the PWIA case both for the inclusive and exclusive conditions. In the studied kinematical region the distortion effects connected with coherent rescatterings of the struck proton dominate at $\vec{p}_m \lesssim 200$ MeV/c. The contribution from the incoherent rescatterings appears to be important at $\vec{p}_m \gtrsim 200$ MeV/c. Our important finding is that apart from the transverse missing momentum distribution, incoherent rescattering do substantially affect also the longitudinal momentum distributions at high missing momentum. This distortion of longitudinal momentum distribution is of a purely quantum-mechanical origin.

Our calculations show that the forward-backward asymmetry connected with the elastic (diagonal) rescatterings of the struck proton on the spectator nucleons is larger than expected from the CT effects by the factor about 2-4 at $Q^2 \sim 5-10$ GeV^2. In the region $Q^2 \sim 2-3$ GeV^2 the expected role of the CT effects is negligible. Thus our results make it clear that the forward-backward asymmetry practically can not be considered as a clean

signal of the onset CT in the CEBAF kinematical region.

Using computed three-dimensional missing momentum distribution we studied the energy dependence of the nuclear transparency for a few kinematical domains. Our results show that despite the rise of $\sigma_{in}(pN)$, leading to the decreasing integral nuclear transparency, the nuclear transparency for the kinematical domains with $\vec{p}_m \lesssim 250$ MeV/c even slightly increases with Q^2. For the first time we performed the comparison of the Glauber model prediction with the recent data from NE18 experiment [8] accurately taking into account the kinematical restrictions in the missing momentum. The energy dependence obtained in the present paper is close to the one observed in [8]. Our detailed calculations of distortion effects also allowed to highlight a limited applicability of treatments of FSI effects based on the conventional DWIA and local-density approximations.

Our important observation is that in the case of $(e, e'p)$ reaction the Glauber formalism is incomplete at sufficiently high longitudinal missing momenta. We have shown that the standard Glauber model Ansatz for the attenuation factor leads to the anomalously slow decrease ($\propto |p_{m,z}|^{-2}$) of the missing momentum distribution at high longitudinal missing momenta. Such a tail is an artifact of neglecting the finite longitudinal size of the region where the interaction of the struck proton with the spectator nucleon takes place. Taking into account of the finite interaction size must drastically change the Glauber model predictions at $|p_{m,z}| \gtrsim 500$ MeV/c. We checked that corrections to the predictions of the standard Glauber approach are still negligible at $|p_{m,z}| \lesssim 300$ MeV/c. It is important that the same incompleteness is inherent to, and persists, also in the color transparency regime at high Q^2, where the Glauber theory must be complemented by the off-diagonal transitions.

The sensitivity of the FSI effects to the finite longitudinal size of the interaction zone for pN collision has important implications for interpretation of the experimental data on $(e, e'p)$ scattering in terms of the short range NN correlations in nuclei. Specifically, it makes it clear that besides the short range NN correlation the measured missing momentum distribution at high missing momenta is sensitive to the nucleon structure as well.

The results of the detailed analysis of the influence of the finite nucleon size upon the missing momentum distribution at high missing momenta will be presented elsewhere.

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Figure captions:

Fig. 1 - The Q^2-dependence of nuclear transparency for $^{16}O(e,e'p)$ and $^{40}Ca(e,e'p)$ scattering. The long-dashed curve is for the inclusive, $(p_{m\perp}, p_{m,z})$-integrated transparency T_A^{inc} as given by Eq. (47), the dot-dashed curve is for the exclusive transprarency as given by Eq. (48).

Fig. 2 - Angular dependence of the inclusive missing momentum distribution $n_{eff}(p_m,\theta)$ in $^{16}O(e,e'p)$ scattering, calculated for FSI without (the left hand boxes) and including (the right hand boxes) the $\Gamma^*\Gamma$ terms, as compared to the single-particle momentum distribution $n_F(p_m)$: the short-dashed, solid, dot-dashed and long-dashed curves are for missing momentum $p_m = 150,\ 200,\ 250$ and 300 MeV/c, respectively.

Fig. 3 - The same as Fig. 2, but for the $^{40}Ca(e,e'p)$ scattering.

Fig. 4 - The missing momentum dependence of nuclear transparency in parallel kinematics ($\theta = 0^o,\ 180^o$) and transverse kinematics ($\theta = 90^o$) calculated (solid curve) for full FSI, including and (long-dashed curve) in the optical approximation, not including the $\Gamma^*\Gamma$ terms.

Fig. 5 - Nuclear transparency for $^{40}Ca(e,e'p)$ scattering as a function of the transverse missing momentum $p_{m\perp}$ at different fixed values of the longitudinal missing momentum $p_{m,z}$. The boxes (**a**), (**c**) are for the full FSI including the $\Gamma^*\Gamma$ terms, the boxes (**b**), (**d**) are for the optical approximation to FSI, with the $\Gamma^*\Gamma$ terms not included.

Fig. 6 - Multiple-elastic rescattering decomposition of nuclear transparency in $^{16}O(e,e'p)$ scattering, integrated over the longitudinal missing momentum. The dot-dashed curve is for the exclusive transparency ($\nu = 0$), the solid curve shows the inclusive transparency summed over all rescatterings (all ν), the dashed curve shows the effect of including the first elastic rescattering ($\nu = 0,\ 1$).

Fig. 7 - The same as Fig. 6, but for $^{40}Ca(e,e'p)$ scattering.

Fig. 8 - The longitudinal-missing momentum dependence of nuclear transparency integrated over the transverse missing momentum $p_{m\perp}$. The solid curves are for the full FSI including the $\Gamma^{*}\Gamma$ terms, the long-dashed curves are for the optical approximation to FSI, with the $\Gamma^{*}\Gamma$ terms not included.

Fig. 9 - The Q^2-dependence of nuclear transparency for $^{16}O(e,e'p)$ scattering at different windows D in the transverse $p_{m\perp}$ and longitudinal $p_{m,z}$ missing momentum in comparison with (the long-dashed curve) the inclusive transparency T_A^{inc} and (the dot-dashed curve) the exclusive transprarency. The solid curve shows transparency $T_A(D)$, calculated with full treatment of FSI ($\Gamma^{*}\Gamma$ terms included), for the $(p_{m\perp}, p_{m,z})$-window D as shown in the corresponding box. The short-dashed curve is the same as the solid curve, but for the optical approximation description of FSI ($\Gamma^{*}\Gamma$ terms not included).

Fig. 10 - The same as Fig. 9, but for the $^{40}Ca(e,e'p)$ scattering.

Fig. 11 - Predictions of nuclear transparency for the missing momentum window ($p_{m\perp} <$ 250 Mev/c,$|p_{m,z}| <$ 50 Mev/c) in comparison with the NE18 determinations for ^{12}C (solid curve) and ^{56}Fe (dot-dashed curve) nuclei. For the ^{12}C nucleus we also show the effect of color transparency (dashed curve) as evaluated in [31].

Fig. 12 - The transverse missing-momentum dependence of nuclear transparency integrated over the longitudinal missing momentum $p_{m,z}$, for the full shell-model calculation with the $\Gamma^{*}\Gamma$ terms included (solid curve) and not included (long-dashed curve), and for the local-density approximation with the $\Gamma^{*}\Gamma$ terms included (short-dashed curve) and not included (dot-dashed curve).

The bottom boxes show the same curves in the blown-up scale.

Fig. 13 - Nuclear transparency in $^{16}O(e,e'p)$ scattering in transverse kinematics $p_{m,z} = 0$ for the full shell-model calculation with the $\Gamma^{*}\Gamma$ terms included (solid curve) and not included (long-dashed curve), and for the local-density approximation with the

$\Gamma^*\Gamma$ terms included (short-dashed curve) and not included (dot-dashed curve).

Fig. 14 - The same as Fig. 13, but for $^{40}Ca(e,e'p)$ scattering.

Fig. 15 - Nuclear transparency in $^{16}O(e,e'p)$ scattering in parallel kinematics $p_{m\perp}=0$ for the full shell-model calculation with the $\Gamma^*\Gamma$ terms included (solid curve) and not included (long-dashed curve), and for the local-density approximation with the $\Gamma^*\Gamma$ terms included (short-dashed curve) and not included (dot-dashed curve).

Fig. 16 - The same as Fig. 15, but for $^{40}Ca(e,e'p)$ scattering.